International Journal of Advanced Research in Engineering Technology & Science
ISSN: 2349-2819
Email: editor@ijarets.org Volume-8, Issue-11 November- 2021 www.ijarets.org

INVESTIGATION OF LINEAR STABILITY OF FLUID FLOWS THROUGH POROUS MEDIA

Prachina Arvind Kumar Patel
Research Scholar
Malwanchal University Indore (M.P.)

ABSTRACT

Stability is the most important property to be possessed by all kinds of systems. The given system may be asymptotically stable (or) aperiodically stable; with regard to a given operating point, it may be relatively stable. For a given linear time-invariant continuous as well as discrete system, numerous algebraic and graphical procedures are available to analyze the stability utilizing the system characteristic equation. Firstly, three simple procedures are presented employing Pseudo Routh Column Polynomial and Auxiliary Polynomials, extracted suitably from Routh table as well as original characteristic equation of linear time-invariant continuous system. Design of parameters residing in the system characteristic equation is also carried out with the help of these polynomials. Secondly, the asymptotic stability and aperiodic stability are dealt for certain class of artificial neural network systems. Main matrices and vertex matrices of these systems are used to develop the discrete characteristic equations and in turn, analyzed for stability employing Marden and Fuller tables. The proposed procedures are direct and simple in approach. Thirdly, certain classes of fuzzy systems represented either in the form of linearized discrete system matrix (or) in the form of discrete relation matrix are considered for stability analysis. In first form, the characteristic equation is used along with Marden and Fuller table for observing asymptotic stability as well as aperiodic stability.

Key words : Stability, relation, network systems

INTRODUCTION

Qin and Kaloni (2012) have considered a thermal stability problem in a rotating micropolar fluid. They found that, depending upon the values of various micropolar parameters and the low values of the Taylor number, the rotation has a stabilizing effect. Sharma and Kumar (2014, 1998) investigated the effect of rotation on thermal convection in micropolar fluids in continuous as well as a porous medium. Sunil et al. (2000) have studied the stability of a layer of couple stress fluid saturating a porous media heated from below in the presence of rotation. Reena and Rana (2008) have studied the thermosolutal convection of micropolar fluids in porous medium in the presence of rotation and established the stabilizing effect of rotation. Vaneeshree and Siddheshwar (2010) studied the effect of rotation on thermal convection in an anisotropic porous medium with temperature dependent viscosity. Bhadauria et al. (2011) studied the thermal instability in a rotating anisotropic porous layer saturated by a nano fluid.

STABILITY THEORY

For carrying out the stability study, suitable mathematical model of a given system is needed. Any non-linear system can be linearized around an operating point and the stability study can be performed. For analysis and design purpose, the stability can be classified into absolute stability and relative stability. The concept of stability of a state of a physical or mathematical system was understood in the eighteenth century and Clerk Maxwell expressed the qualitative concept clearly in the nineteenth century: "When…an infinitely small variations of the present state will alter by an infinitely small quantity in the state at some future time, the condition of the system, whether at rest or in motion, is said to be stable; but when an infinitely small variation in the present state may bring about a finite difference in the state of the system in a finite time, the condition of the system is said to be unstable."

The study of stability in fluid flows aims at understanding the abrupt changes which are observed in fluid motion as the external parameters are varied. The theory of stability may be stated as follows: "Given an equilibrium state of a physical system, whose stability we wish to study, we consider a state near equilibrium and ask whether in the course of time the system will tend towards the given equilibrium state."

The mathematical formulation of the stability theory proceeds from the non-linear partial differential equations. The unknown quantities are functions of three space coordinates and time, and are subjected to some boundary conditions. Certain special permanent type solutions of such general problems, called the basic solutions, are usually of particular interest. The linear stability theory for a particular flow starts with a solution of the equations of motion representing the flow. One assumes that the disturbances are small so that the equations governing the disturbances may be linearized, i.e. the terms quadratic or higher in the disturbances and their derivatives may be neglected. Whether a small disturbance that is superimposed will be amplified or damped, depends on the properties of the fluid, the pattern of the primary flow and the nature of the disturbances. If all such disturbances are damped, the flow is stable, otherwise it is unstable even if certain particular disturbance does vanish with time. Therefore the behavior of all possible disturbances is investigated as time passes. Thus, instability is characterized by the existence of perturbations which grow without limit in course of time. In the case of stability all perturbations tend to zero as time tends to infinity. A limiting case between stability and instability i.e. neutral stability, occurs, when there exists a perturbation, which after being introduced remains of essentially constant amplitude in the course of time, while all other possible perturbations tend to zero. If at the onset of instability a stationary pattern of motions prevails, then one says that the Principle of Exchange of Stabilities (PES) is valid at the marginal state and if at the onset of instability oscillatory motion prevails then we have a case of overstability.

The stability of a given flow is usually analyzed through **Normal Mode Technique**. This technique has been widely and successfully used in the linear theory of stability. The stability problem involves the growth rate of an arbitrary infinitesimal perturbation and the stability means the stability with respect to all such possible disturbances. By Normal Mode Technique, for the stability of a hydrodynamical system, the linearized perturbation equations are set up first in a single perturbation variable by eliminating the remaining variables from the linear equations derived from the equations of conservation of mass, momentum and energy by retaining only the linear terms in perturbed quantities. These equations are then solved either analytically with the help of variation procedure or through an integral equation under a set of appropriate boundary conditions which leads to the dispersion relation in the parameter determining the stability of a system. Therefore, to determine the effect of a particular physical parameter on the growth rates, we analyze the change of varying that parameter while keeping the other parameters fixed. An increase in growth rate implies the destabilizing influence of that particular parameter and a decrease in growth rate shows stabilizing influence of the parameter.

For the investigation in any stability analysis to be complete, it is assumed that the perturbations can be resolved into dynamically independent wave like components, each component satisfying the linearized equations of motion and the boundary conditions separately. Disturbances in all the cases must be expended in all possible forms of time function constituting the time behavior of the quantities in the system. Thus, if $f'(x,y,z,t)$ is a typical wave like disturbance, we expand it in the manner $f'(x,y,z,t)=f(z)\exp\{ik_x x+ik_y y+\sigma t\}$, where $f(z)$ is the amplitude of disturbance, k_x and k_y are the wave numbers in x and y directions, respectively and $k=\sqrt{k_x^2+k_y^2}$ is the real wave number of the disturbance and σ is a constant to be determined and, in general, is a complex constant. Since the perturbation equations are linear, the reaction of the system to a general disturbance can be determined if we know the reaction of the system to disturbances of all assigned wave numbers. In particular, the stability of the system will depend on its stability to disturbances of all wave numbers and instability will follow from the instability with respect to even one wave number.

The assumption that a disturbance can be represented by wave components, according to the method of normal modes, serves to separate the variables and reduces the linearized equation of motion from partial to ordinary differential equations. The final process consists of solving the set of coupled, homogeneous, ordinary linearized differential equations governing the amplitude $f(z)$, subject to appropriate boundary conditions of the problem under investigation. Indeed, the requirement that the equation allow a non-trivial solution satisfying the various boundary conditions leads directly to a characteristic value problem in σ. In general, the characteristic value of σ be complex, whose real and imaginary parts will, apart from various modes, depend upon the physically significant parameters involve in the system.

REVIEW OF LITERATURE

Moreau and Aeyels (2000) have introduced a notion of practical stability for dynamical systems depending on a small parameter. Stabilization of linear systems was proposed by Elia and Mitter (2001), based on the idea of robust Lyapunov functions. In this state-feedback controller and estimator have been derived using quantizers. A Linear Matrix Inequality (LMI) approach has been developed by Ni and Joo Er (2002) for stability of linear systems with delayed perturbations.

Kharitonov and Niculescu (2003) have worked with the stability of some class of linear delay perturbed systems under the assumption that the nominal system is asymptotically stable. The stabilization of systems that switch among a finite set of controllable linear systems with arbitrary switching frequency has been developed by Cheng et al (2005). Sano and Kunimatsu (2005) treated a linear bioprocess model with recycle loop and analyzed the spectrum of closed-loop operator in order to discuss stability. Tan et al (2006) analyzed the stability of TCP/ RED systems in AQM routers by applying the time-delay control theory.

Leung et al (1997) examined the stability and statistical properties of second-order bidirectional associative memory (BAM). By using the method of Lyapunov functional stability analysis of bidirectional associative memory networks with time delays was studied by Feng and Plamondon (2003).

Suykens et al (2000) derived a condition for robust local stability of multilayer recurrent neural networks with two hidden layers. Here, the derived criterion for robust local stability has been applied in order to constrain the dynamic back propagation procedure for multilayer recurrent neural networks. Barabanov and Prokhorov (2002) performed stability analysis of a discrete time recurrent neural networks. In this, a simple statespace transformation was devised to convert the original recurrent neural network equations to a form suitable for stability analysis. Liu and Han (2006) developed stability criteria of recurrent neural networks using Volterra Integro- Differential equations.

CLASSIFICATION OF FLUIDS

Fluid mechanics can be divided into fluid statics, the study of fluids at rest; fluid kinematics, the study of fluids in motion; and fluid dynamics, the study of the effect of forces on fluid motion. Fluid mechanics concerns itself with the investigation of motion and equilibrium of fluids.

We normally recognize three states of matter: solid, liquid and gas. However, liquid and gas are both fluids: in contrast to solids they lack the ability to resist deformation. Because a fluid cannot resist the deformation force, it moves, it flows under the action of the force. Its shape will change continuously as long as the force is applied. A solid can resist a deformation force while at rest, this force may cause some displacement but the solid does not continue to move indefinitely. Thus, a fluid is a substance that does not has characteristic shape or extensive physical property such as crystalline structure. Also, a fluid is a substance changed by an amount, which is not so small as compared to suitable chosen forces, however small in magnitude. From a morning coffee to an evening bath, fluids are all around us. Water is a fluid and so is air. In space and inside stars there is also another kind of fluid called plasma.

NON-NEWTONIAN FLUIDS

Any fluid that does not obey the Newtonian relationship between the shear stress and shear rate is called non-Newtonian. High molecular weight liquids which include molten polymers and solutions of polymers, as well as liquids in which fine particles are suspended (slurries and pastes), are usually non-Newtonian. Liquid metals also exhibit non-Newtonian flow characteristics. The atomic mass and density of liquid metals is quite large compared with fluids that have simple Newtonian viscosity behavior. Common examples include mayonnaise, peanut butter, toothpaste, egg whites, liquid soaps and multi-grade engine oils.

MICROPOLAR FLUID

Although the non-Newtonian behavior of many fluids has been recognized for a long time, the science of rheology is, in many respects, still in its infancy, and new phenomena are constantly being discovered and new theories proposed. Advancements in computational techniques are making possible much more detailed analyses of complex flows and more sophisticated simulations of the structural and molecular behavior that gives rise to non-Newtonian behavior. Engineers, chemists, physicists, and mathematicians are actively pursuing research in rheology, particularly as more technologically important materials are found to display non-Newtonian behavior. There are some more types of important non-Newtonian fluids other than above mentioned fluids. One of them is micropolar fluid.

Micropolar fluids are fluids with microstructure belonging to a class of non-Newtonian fluids with non-symmetrical stress tensor. Micropolar fluids have the micro-rotational effects and micro-inertia effects. The concept of micro-rotation was initially proposed by Cosserat and Cosserat (1909)[9] which was applied successfully to describe the flow of fluids with micro-structures [Condiff and Dahler (1964)[10]]. Inspired by them, Eringen (1964)[11] analyzed a new class of fluids called “micro fluids” exhibiting micro effects similar to simple micro-elastic materials. In these fluids local structures and micro-rotations of the material particles contained in each of its volume element i.e. gyration effects play an important role. The stresses and stress moments are functions of
deformation rate tensors and various micro-deformation rate tensors and hence these types of fluids are quite complicated to the extent that even in the simplest case of constitutive linear theory, these contain 22 viscosity coefficients. Therefore it is not easily amenable to construct and analyze the mathematical models of such type of problems. Eringen (1966)[12] introduced a subclass of these fluids called micropolar fluids, in which micro-rotational effects like micro-rotational inertia are important and included in the analysis but micro stress stretch of the particles is not allowed. These fluids can support the couple stress, the body couples, the non-symmetric stress tensor and a rotation field independent of velocity field. The theory, thus, has two independent kinetics variables: the velocity vector and the spin or micro-rotation velocity vector. This theory involves only four additional viscosity coefficients, one introduced through the linear constitutive equation for a non-symmetric stress tensor and other three due to linear constitutive equation for a couple stress, hence converts to a simple, elegant and realistic theory.

CONCEPT OF POROUS MEDIUM

A porous medium is a material containing pores (voids). The skeletal portion of the material is often called the "matrix" or "frame" which is usually a solid, but structures like foams are often usefully analyzed using the concept of porous media. It is however much more difficult to give an exact geometrical definition of what is meant by the notion of a pore. Intuitively pores are void spaces, imbedded in a material, may be either connected or isolated (non-connected), distributed more or less frequently in either a regular or random manner in a material. These pores may be effective or ineffective. Effective pore mean the pores through which the fluid can actually pass. These pores contribute towards the porosity of the material. By ineffective pores we mean the pores through which the fluid can't pass. This may be either due to surface-tension caused by fine holes or the pores may not be interconnected, so that they do not affect the flow directly but may affect the compressibility of a medium.

The voids in a porous medium may be classified according to the behavior of the fluid within these spaces. The smallest void spaces in which molecular forces between the molecules of the solid and those of the fluid are significant are classed as interstices or capillaries. The largest void spaces in which the motion of a liquid is partially affected by the walls of the voids are referred to as caverns. The spaces which are intermediate in size between capillaries and caverns are referred to as pores. It is this effect of the minute openings that definitely differentiates this subject from that of the usual hydrodynamics or hydraulics.

While the ordinary pipe or capillary considered in hydraulics is equivalent to a single series of connected pores, the channel composed of porous material involves a very large number of such elements with multiple lateral connections. As a particular case in a capillary tube or pipe carrying a fluid in viscous flow, the velocity is not uniform across the section but has a parabolic distribution with a maximum velocity at the center of the channel. While in a linear system consisting of a porous medium the velocity distribution across a single pore opening will have similar characteristics, the velocity, when considered macroscopically across the whole channel will be uniform.

A porous medium is not restricted to have the pores belonging to one class. One porous medium may be imbedded with the pores of different sizes and shapes. In the simplest situation ("single-phase flow") the void is saturated by a single fluid. In "two-phase flow" a liquid and a gas share the void space. In a natural porous medium the distribution of pores with respect to shape and size is irregular. According to this description the term porous medium may encompass a very wide variety of substances. Some of the examples of porous media are tower packed with pebbles, Berl Saddles, beach sand, granules, catalyst pellets, wood, woolen packing, leather, foamed plastic, human lung, column packing, soil, naturally occurring rocks, concrete, cement, bricks, paper cloth, filter paper, rye bread etc..

CONCLUSION

Microscopic property is one that characterizes the structure of the pores while the macroscopic description is a description of media in terms of the average, or bulk properties and their variations at sizes or scales much larger than pores. Almost all porous media whether natural or artificial have a random void structure. At present, it is quite difficult rather impossible to describe a porous medium by the description of the sizes, shapes, orientations and interconnections of voids of the medium, it will be worthwhile to characterize a porous medium by two of its static properties named porosity and permeability as consideration of these replaces all the complexities of the porous medium.

REFERENCES

- Chabra R. P. and Richardson J. F., Non-Newtonian Flow and Applied Rheology: Engineering Applications, 2nd Edn., Oxford, UK (2008).
- Perry R. H. and Green D. W. (Eds.), Perry's Chemical Engineer's Handbook, 7th Edn., McGraw-Hill, New York (1997).

- Baird D. G., Flow Behavior of Viscoelastic Fluids, McGraw-Hill, New York (2011).
- Rivlin R. S. and Ericksen J. L., Stress-Deformation Relations for Isotropic Materials, *Journal of Rational Mechanics and Analysis*, 4, 323 (2004).
- Walters K., Non-Newtonian Effects in Some Elastico-Viscous Liquids whose Behavior at Small Rates of Shear is Characterized by a General Linear Equation of State, *Quarterly Journal of Mechanics and Applied Mathematics*, 15, 76 (2008).
- Walters K., The Solution of Flow Problems in the Case of Materials with Memory, *Journal of Mechanics*, 1, 469 (2010).
- Condiff D. W. and Dahler J. S., Fluid Mechanical Aspects of Antisymmetric Stress, *Physics of Fluids*, 7, 842 (2010).
- Eringen A. C., Simple Microfluids, *International Journal of Engineering Science*, 2, 205 (2005).
- Eringen A. C., Theory of Micropolar Fluids, *Journal of Mathematics and Mechanics*, 16, 1 (2001).